The Cost of Anarchy in Self-Commitment Based Electricity Markets

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Abstract

With the advent of restructured electricity markets a contentious market design issue has been whether unit commitment decisions should be made centrally by the system operator or by individual generators. Although a centrally committed market can, in theory, determine the most efficient commitment, they have been shown to suffer some equity and incentive problems. A self-committed market can overcome some of these incentive issues, but will generally suffer efficiency losses from not properly coordinating commitment and dispatch decisions between individual generators. This paper examines the issue of dispatch efficiency raised by the design of markets based on central versus self-commitment by determining a set of ‘competitive benchmarks’ for the two market designs. Comparing the total commitment and dispatch costs of the two markets provides a bound on the productive efficiency losses of a self-committed market.

Key words: Unit commitment, market design, incentives, imperfect competition

1. Introduction

The introduction of competition in the electric supply industry has led to a number of important questions regarding the need for organized markets to efficiently and reliably coordinate the power system, and the desirable features and scope of those markets. Complicating electricity market design, power systems are subject to a number of ‘network’ constraints, in that these constraints depend on the actions of every market participant and each participant can impose an externality on others using the power system. That is, the ability of customer A to purchase power from generator B can depend on the actions of generator C or of consumer D. These complexities have called into question the ability of decentralized markets to efficiently and feasibly commit and dispatch units while respecting power system constraints and the market design rules for any centralized markets operated by a system operator (SO). While a centralized market can, in theory, find the most efficient dispatch of the generators, the market designs suffer equity and incentive problems. Decentralized designs can overcome some of these issues but will suffer efficiency losses due to the loss of coordination between market participants.

As electricity markets in various countries have been restructured and have evolved, different approaches have been used with varying degrees of success. In the US, for example, the move towards standard market design has led to heavy reliance on open and transparent centralized markets. The British market, by

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contrast, started with a centralized market in the original Electricity Pool and moved to a more decentralized design under the New Electricity Trading Arrangements (NETA) and subsequent reforms under the British Electricity Trading and Transmission Agreements (BETTA), which were meant to overcome some of the problems experienced under the original centralized pool design. Although these design differences are driven by realities of the market such as asset ownership, generation mix, and system infrastructure, different ‘philosophies’ regarding the proper role of centralized markets have also played a role in determining the scope of any centralized markets. For example, O’Neill et al. (2008); Moran and Skinner (2008) advocate centralized versus decentralized markets, respectively, based on these philosophies and experiences in different markets.

This paper revisits the issue of dispatch efficiency raised by the design of markets based on centralized versus decentralized dispatch. Using actual market data from an SO, a set of ‘competitive benchmarks’ for a centralized and decentralized market are computed and compared. These competitive benchmarks assume that generators do not behave strategically in manipulating their offers in the two markets. Comparing the total commitment and dispatch cost of the two market designs shows the extent of productive efficiency losses from a decentralized market, which are small but non-trivial. Comparing total settlements costs of the two designs shows that generators can extract significantly higher payoffs from consumers under a decentralized design, suggesting that this design may be the wrong approach if the goal of restructuring is to reduce consumer costs. These higher prices under a decentralized design would further result in allocative efficiency losses in markets with demand response.

The remainder of this paper is organized as follows: section 2 provides the context on the debate regarding the scope of centralized markets. Sections 3.1 and 3.2 further describe centrally and self-committed markets and the market models used in the simulations, with section 3.3 providing the results and comparison of the two designs. The technical appendix outlines the formulations and algorithms used in the simulations and analysis. Section 4 concludes.

2. Centralized Versus Decentralized Markets

Unlike other commodities, electricity has a number of technical constraints, which must be obeyed to ensure feasible and reliable service. Complicating electricity market design, power systems are subject to physical constraints, which make markets for electricity inherently incomplete. This incompleteness is due in part to costly storage of electricity, complex power flows within the transmission network, extremely price-inelastic short-run demand for energy, and the need to constantly balance supply and load in real-time.

Injections and withdrawals of electricity cannot be directed along a specified path within a transmission network, rather power flows along each line in inverse proportion to the line’s impedance. Flows on lines within the network are constrained by physical limitations and environmental factors. Moreover, allowing an injection of power, which will flow in a constrained (also known as congested) direction along a path, may be contingent upon another injection, which provides ‘counterflows’ to relieve congestion on that path. These characteristics of power systems mean that use of the transmission network for injection and withdrawal of energy must be properly coordinated, otherwise the resulting dispatch may not be simultaneously feasible or optimal.

Generating units are constrained in the time it takes them to startup or shutdown, and the rate at which they can adjust their output. Thermal units have non-zero minimum generating constraints and ‘forbidden zones,’ in which they cannot operate stably, when they are online. Other types of generating units, such as combined-cycle gas turbines (CCGT) and cascaded watershed hydroelectric systems tend to have complex constraints restricting their operation. Due to the stochastic nature of demand fluctuations, generators must be able to adjust their real and reactive power outputs in real-time to ensure constant load balance. Other random contingencies such as transmission equipment failures or forced generator outages may also require generators to adjust their outputs within a short period of time to maintain system reliability. Thus,

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1 As in any market with price-elastic demand, prices which are above the competitive level will result in higher producer profits at the cost of consumer welfare losses.
efficient and reliable operation of the system requires having a sufficient number of generators online and available to react to variations in load and other contingencies at least cost.

These physical realities of power systems have given rise to a debate about whether competitive pressures in a decentralized or bilateral market can lead to efficiency and reliability of the system, or if we must rely on centrally organized markets. Whatever market design is ultimately adopted, there are several objectives it should address, including:

- **Efficient Dispatch.** The market should ensure that generators within the system are efficiently dispatched to meet system load at least cost, while satisfying the requisite operational constraints;

- **Network Feasibility.** The pattern of injections and withdrawals within the transmission network and the resulting power flows must be feasible; and

- **System Reliability.** The market must ensure that there is sufficient excess generating capacity online and available (known as ancillary services) to react to contingencies such as load fluctuations, or generator or transmission outages within a sufficiently short amount of time to ensure reliability.

This market design debate has often been framed as being between the two extremes of a bilateral versus Poolco model, as described by Hogan (1994).

The bilateral model emphasizes direct transactions between buyers and sellers. It argues that competitive forces in the marketplace will resolve any infeasibilities in dispatches and schedules and deliver efficient and reliable electric service, without the need to design or impose new market institutions. Technical difficulties such as load balance, power flow constraints, and ancillary services would be resolved by market transactions. According to proponents of the bilateral model, competitive forces would minimize costs while capturing any potential efficiency gains, desirable features that may be hampered by the rigidities of centralized markets.

The Poolco model, on the other hand, emphasizes the need for tight coordination of the power system to ensure efficient, feasible, and reliable service. This model has influenced the Standard Market Design (SMD), “blessed” by the Federal Regulatory Energy Commission in the US and variants of this model have been implemented (or implementation is underway) in about 50% of the US states. Proponents of the Poolco model have argued that power balance, provision of ancillary services, and feasible power flows and counterflows within the transmission network may not be easily achieved through bilateral transactions. Moreover, the system must be in continuous real-time balance to ensure reliable operation, meaning the bilateral model may have to rely on a series of short-run minute-by-minute bilateral transactions to constantly adjust schedules and ensure system feasibility. Hogan (1994), for example, paints an extreme picture of a ‘strict bilateral market,’ in which the market is completely reliant on bilateral transactions without any centralized markets. His example envisions that consumers\(^3\) must contract with generators to provide energy and other essential services, and manage these contracts in real-time independently of one another. In his example, each customer and generator would have to track changes in one another’s output and consumption, and make adjustments to their own generation and load or make spot bilateral transactions to correct load imbalances. Furthermore, the two parties would have to adjust their portfolio of transmission capacity rights to enable the actual real-time power flows.

Much of this debate originally centered on use of the transmission system. Ruff (1994) advocates development of organized spot markets, in which energy and transmission are traded in an integrated fashion, as opposed to relying on bilateral trading between the incumbent utilities and new entrants. He argues that absent organized spot markets the industry may fall back on traditional wheeling agreements, which are ad hoc trades that were used by vertically integrated utilities under the regulated industry paradigm to handle a few incremental trades with other utilities. These wheeling agreements would be, in his opinion, inadequate to deliver all the potential efficiency gains and cost savings from trading in a restructured wholesale market.

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2 Joskow and Schmalensee (1983); Hunt (2002) provide a more thorough discussion of important considerations in electricity market design.

3 The term ‘consumer’ is used throughout this paper to encompass any entity purchasing in the wholesale market. This includes both large customers purchasing electricity for themselves as well as load-serving entities.
Hogan (1995) further advocates a centralized Poolco model. He argues that the physical realities of power flows would make the allocation of physical transmission rights to parties difficult, and as such a bilateral approach may not fully realize efficient trade. In his opinion, only a centralized market which co-optimizes generation and transmission flows will achieve least-cost dispatch.

Although much of the debate initially focused on use of the transmission network, another important facet of this market design issue is how the commitment (on/off) status of generators should be determined. Under the traditional regulated monopoly paradigm, utilities would determine unit commitments to minimize total costs while meeting load, ancillary service, transmission, and generator operational constraints over a fixed planning period, typically between 24 to 168 hours. One of the difficulties that has plagued the solution of unit commitment problems is their computational complexity. Because commitment decisions are binary (i.e. a generator is either on or off in a given hour), they must be formulated as mixed-integer programs (MIPs), which are computationally demanding as shown by Guan et al. (2003) and others. As such, utilities have had to rely on approximation algorithms such as Lagrangian relaxation (LR) to find a near- (but sub-) optimal feasible solution within a reasonable amount of time, so the utility can follow the least-cost commitment.

In a competitive setting, the market must provide for this unit commitment planning. The Poolco model typically envisions unit commitments being determined in a centralized manner by the SO. The SO solicits generation offers from generators, which specify their operating constraints and cost structure and combines these with hourly load forecasts to determine a least-cost commitment and dispatch, which satisfies the requisite generator and system constraints. Ancillary service constraints are normally based on possible generator and transmission contingencies. For instance, it is typical to ensure that there is sufficient excess generating capacity available to react to the forced outage of the largest and second largest generators within a set number of minutes. This unit commitment process is typically done day-ahead, meaning commitments are determined for the following day and an advisory schedule is given to each generator specifying their generation in each hour. On the following day once the commitments are fixed, the SO will then redispatch generators in real-time to meet the actual load and obey real-time system constraints. This allows the dispatch to react to contingencies such as load shocks, forced generator outages, or changes in transmission network topology. Because most generators incur fixed costs when starting up and when online, the cost offers generators submit to the SO typically consist of three parts:

1. a startup cost, which is incurred whenever the unit is started from an offline state;
2. a no-load cost, which is the spinning cost a generator incurs from being online, regardless of its actual generation; and
3. its actual variable generation costs, which are typically expressed as a non-decreasing step- or piecewise linear-function.

The bilateral model, by contrast, leaves these commitment decisions to individual generators. In a purely bilateral market, generators will contract to deliver energy to consumers. This contracted energy will then, in real-time, be supplied by some combination of producing energy from its own generating assets or contracting with another party to deliver the energy. To produce energy itself, a generator will have to determine the commitment status of its own units to ensure it can deliver the energy. Similarly, consumers can contract with generators to hold excess generating capacity for ancillary services to ensure its demand can be reliably served under different system conditions, which will again factor into a generator’s unit commitment decisions. If generators are liable for damages from not serving a customer’s load, the generator may include its own ancillary service requirements in its commitment decision.

4 Muckstadt and Koenig (1977) provided one of the first formulations of unit commitment problems, and proposed the use of the LR algorithm in their solution. Hobbs et al. (2001); O’Neill et al. (2008) provide more recent treatments of unit commitment.
5 Although some authors use the term ‘bid’ to describe the cost and operating constraint data provided to the SO by a generator, this paper calls them ‘offers,’ which distinguishes a generator’s offer to generate energy from a customer’s bid to purchase energy.
6 Some markets with centralized unit commitment have longer planning horizons. The new market design put forth by the California ISO, for example, has a separate unit commitment for the second day out, which is used to advise units with long start times to have themselves ready to be online.
7 Some SOs also allow generators to specify a shutdown cost, which is incurred whenever the unit is halted from an online state. Generators operating in markets which do not accept shutdown costs can roll these shutdown costs into the unit’s startup cost.
Alternately, some bilateral designs call for the SO or another entity to operate energy and ancillary service markets into which generators can offer their generation. These markets are typically operated day-ahead, but in some cases hour-ahead or markets with other timeframes are operated. Although the assignment of generators to generate or provide ancillary services is done in a centralized manner in such a market, generators individually decide whether to commit themselves in expectation of revenues from the market. Wilson (1997), for instance, suggested the use of such a market in the original restructured California market. In a strict bilateral market, real-time load imbalances would have to be resolved directly between contract counterparties, for instance as described by Hogan (1994). Otherwise, some bilateral markets have real-time balancing service markets in which imbalance energy (both incremental and decremental) can be traded.

Besides philosophical differences regarding the extent to which transactions in the market should be standardized and centralized within the SO’s purview, there are also incentive and efficiency issues arising from the two designs. The clear advantage of having the SO make commitment decisions in a centralized manner is that a centralized market will, in theory, find the most efficient commitment and dispatch of units. Generators acting independently of one another may not be able to achieve this, because the non-convex nature of generator operating constraints and costs mean there are efficiencies to be gained from coordination amongst generators. Moreover, a centrally committed market could easily find a feasible schedule, while this may prove more difficult in a decentralized paradigm.

Centrally committed markets do, however, suffer from incentive issues, which call into question the efficiency of the underlying commitment. For one, the auction into which generators submit their operating constraints and costs is not strategy-proof, meaning generators may find it profitable to misstate their parameters. Oren and Ross (2005) demonstrate, by studying simple examples, that generators can profitably misstate their ramping constraints. Because these parameters are then used by the SO in determining the commitment of units, this type of misrepresentation on the part of generators can result in productive efficiency losses. That is, the SO could find a commitment which appears to be least-cost on the basis of the parameters given to it by generators, but which is suboptimal because these parameters are misstated.

Furthermore, Johnson et al. (1997) highlight other equity, incentive, and efficiency issues with centralized unit commitment. They demonstrate that if the SO solves for a commitment using an LR algorithm and makes binding commitment and dispatch decisions on the basis of this solution, different near-optimal solutions which are the same in terms of total commitment costs can result in vastly different payoffs to individual generators. Thus, seemingly benign programming parameters and heuristics used in the LR algorithm could arbitrarily determine which generators are ‘winners’ and ‘losers’ in being committed and receiving energy payments. Generators, knowing the nature of the LR commitments, may be further inclined to misstate their constraint and cost parameters to increase their likelihood of being committed and dispatched, which would again call into question the efficiency of the solution due to incorrect data in the underlying problem.

Although recent advances in computational capabilities and optimization software now make it tractable to solve the MIP formulation of a commercial-scale SO unit commitment problem, Sioshansi et al. (2008) show that eliminating the issues with LR solutions raised by Johnson et al requires solving the MIP to complete- (as opposed to near-) optimality. Otherwise, they demonstrate that if the algorithm used to solve the MIP must be stopped by the SO prematurely, the sub-optimal solution can yield generator payoffs which are different from the optimum. They further demonstrate that because energy prices are determined by the marginal cost of generation in each hour, which is in turn determined by the set of units committed in each hour, these energy prices can deviate significantly from the optimum as well. Moreover, they show that the relative size of these payoff and energy price deviations do not necessarily decrease as the commitment gets closer to optimal—meaning the only way to completely eliminate the issues raised Johnson et al is to solve the MIP to complete optimality. Although some SOs with central commitment have adopted MIP (as opposed to LR) in the solution of their unit commitment problems, none of these SOs are able to solve their unit commitment to complete optimality.

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8 Nonconvexities imply that an efficient Walrasian equilibrium with linear prices may not exist. See O’Neill et al. (2005) for a more detailed discussion.
optimalit y within the limited timeframe they have to determine the next day’s commitments and advise generators, implying the issues raised by Johnson et al have not been alleviated in these markets.

The issues raised by Johnson et al have been used to champion simpler market designs in which unit commitment decisions are made in a decentralized fashion by individual generators. Advocates of decentralized commitment claim it will reduce incentive compatibility issues since generators must internalize their operating constraints while minimizing production costs. Furthermore, if generators are making commitment decisions to maximize profits in expectation of energy prices, this process is similar to solving a centralized unit commitment by means of an LR algorithm.\(^9\) Wilson (1997) proposed a market design with self-commitment and a multi-round energy auction in which generators and loads would iteratively submit one-part offers specifying prices at which they would be willing to supply and consumer energy until prices. The offers would be iteratively updated, subject to some proposed activity rules,\(^10\) until the prices and dispatch converged. His proposal was geared towards the original California market design, and while some of the elements were used, the design eventually settled on a simpler single-round auction.

Given these divergent philosophies on the fundamentals of market designs and the pros and cons of centralized versus decentralized markets, different jurisdictions have implemented a variety of designs spanning the spectrum between the bilateral and Poolco models. Britain, which was one of the first markets to restructure, originally established the centralized Electricity Pool, which was a mandatory market through which all energy was transacted.\(^11\) The pool operated as a day-ahead market which accepted generation offers consisting of three-part costs and unit operation constraints, and determined a least-cost commitment and dispatch of the system. The pool relied on the same software program, GOAL, which was used by the vertically integrated monopolies prior to restructuring to determine these commitments. Because generators had access to the GOAL software, they were able to manipulate and misstate the cost and constraint parameters in their offers in such a way to maximize profits, as opposed to submitting the true values.

A review of the pool, conducted by Offer (1998), further criticized the GOAL software as being opaque in its pricing (due to various heuristics employed in the feasibility phase of the LR algorithm\(^12\)) and using scheduling algorithms that are more appropriate for a vertically integrated utility than for a competitive market. The British market was eventually restructured under NETA, which established a series of overlapping voluntary markets, that rely primarily on bilateral and long-term contracting between generators and consumers. NETA has a minimal balancing market based on a pay-asbid auction three and a half hours prior to physical delivery. NETA was further reformed under BETTA, but still maintained a decentralized design with self-commitment.

The market in Texas, established in 2001 is based almost entirely on individual bilateral transactions, relying on generators to commit their units individually.\(^13\) The bulk of energy and ancillary services (typically 95-98%) are traded and contracted on a long-term basis between generators and consumers, with the SO operating markets for imbalance energy and to acquire ancillary services as a provider of last resort. Day-

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\(^9\)When an LR algorithm is used to solve a central SO unit commitment problem, it works by relaxing load-balance and ancillary-service constraints and penalizing these in the objective function. The problem then decouples in the sense that there are no longer any constraints linking the decision variables of the individual generators, and each can be solved independently of the others. The algorithm then works to find coefficients for the penalty terms which incent sufficient generation and reserves to satisfy the relaxed constraints. These coefficients can, in turn, be thought of as energy and ancillary service prices, and each generator’s decoupled problem can be rewritten as a profit-maximization problem, which would be equivalent to a market with decentralized commitment. The important distinction, however, is that because a centralized unit commitment problem typically has a duality gap, heuristics must be used to ensure primal-feasibility of the dual solution. This heuristic process can commits unit at a net profit loss, meaning that the final feasible centralized solution can differ from the decentralized in terms of some near-marginal commitments and dispatches. See Wolsey (1998) for a further discussion of the LR algorithm.

\(^10\)The purpose of the activity rules is to ensure early price discovery, fast convergence, and to prevent large generators and consumers from manipulating the auction by withholding themselves from the market until the final round of bidding.


\(^12\)A Branch and Bound approach, on the other hand, guarantees feasibility of the solution but nonetheless would require some heuristics to decide when to terminate the search since running the algorithm to completion is typically unrealistic in a practical setting.

\(^13\)The Texas market is currently undergoing a major reform which among other things will establish a voluntary day ahead market with centralized unit commitment based three part offers. See Adib and Zarnikau (2006) for a more detailed discussion of the Texas market.
ahead, generators and consumers submit schedules to the SO specifying expected injections and withdrawals of energy and ancillary service schedules. Originally, market rules required consumers to submit balanced schedules—meaning scheduled injections had to cover expected loads. This balanced schedule requirement was meant to encourage consumers to contract for their expected load as opposed to relying on the balancing market for procurement of base load energy. Eventually, in mid 2002, the balanced schedule requirement was dropped, yet the balancing market still only accounted for a small fraction of energy trades.\(^{14}\) Due to the inaccuracy of load forecasts and because of random events such as generator or transmission outages generators and consumers would have to have a means of making real-time adjustments to their schedules. As opposed to requiring generators and consumers to make these adjustments individually, the SO operates a balancing energy services (BES) market. Generators can submit offers for both incremental and decremental energy, which is then dispatched in real-time to ensure load balance, but the offers are simple one-part energy-only offers specifying a price at which a generator is willing to increase or decrease its generation from its schedule. The BES market is further used by the SO to purchase imbalance energy in order to relieve congestion on transmission lines. Because of the highly decentralized nature of the Texas market (with the exception of the pool-like BES market), there is no centralized mechanism by which commitment decisions are made. Rather, generators individually determine commitments for their units in order to meet their contractual obligations and any imbalance or ancillary service\(^{15}\) sales they expect to make.

The two examples presented are meant to be illustrative of centrally\(^ {16}\) versus self-committed markets, with other markets falling between these two examples. Table 1 summarizes where other major restructured markets fall with regard to determining unit commitments. The Australian National Electricity Market, original California market, and NordPool are further examples of self-committed markets. The original California market is slightly unique in that it is a hybrid of a Poolco model with self-commitment. Generators would make commitment decisions individually, but nearly all energy was traded day-ahead in a mandatory central power pool. The three large incumbent utilities were initially prevented, by regulatory mandate, from contracting for energy on a long-term forward basis.\(^ {17}\) This restriction was later relaxed but the utility had no incentive to contract since the regulator would not guarantee that the contract cost would be considered “prudent” if it turned out to be higher than the average spot price on the central power exchange. ISO New England, PJM, the proposed California market redesign, and the Texas market redesign are examples of centrally committed markets.

3. Comparison of Centrally and Self-Committed Markets

Clearly centrally and self-committed markets present tradeoffs, which must be evaluated in addressing market design issues. Centrally committed markets strive for the least-cost commitment and dispatch of generators by solving for a commitment which minimizes the SO’s cost objective. Theoretical studies and empirical observations have demonstrated incentive and equity issues with centralized unit commitment, which call into question the efficiency of the central solution, however. Self-commitment has been offered as a viable alternative, which addresses and reduces some of the issues with centrally committed markets. Self-commitment will, however, suffer from some loss of coordination amongst generators, resulting in efficiency losses. Thus, central and self-commitment are two imperfect market models with inherent shortcomings.\(^ {19}\)

\(^{14}\)See Hortaçsu and Puller (2008) for a discussion of the implications of the balanced schedule requirement.

\(^{15}\)The ancillary service market operates similarly to the BES, into which generators submit offers specifying prices at which they are willing to be held in reserve for different qualities (i.e. reaction times) of ancillary services.

\(^{16}\)Most centrally committed markets do allow for self-commitment.

\(^{17}\)The one exception to this was a short-lived and highly illiquid block-forward market which the California Power Exchange (the entity which administered the day-ahead spot market) operated.

\(^{19}\)In theory, a Vickery-Clarke-Groves (VCG) mechanism (assuming the SO’s unit commitment problem could be solved to complete optimality) would address both the incentive and efficiency issue. This suffers the obvious shortcoming that the SO unit commitment cannot be solved to optimality. In fact, the VCG mechanism requires solving the unit commitment problem multiple times—once for each generation firm in the market. Moreover, VCG payments are discriminatory, complicated, and not budget balanced making the mechanism an unrealistic option. See Mas-Colell et al. (1995) for a discussion of mechanism design and the VCG auction.
To compare the two designs, this section describes and analyzes models of the two markets. The models assume that the markets are competitive—thereby eliminating the incentive issues and focusing instead on the relative efficiency losses and settlement costs of relying on self as opposed to central commitment.

Unit commitment refers to determining a short-term schedule of the on/off status of generating units to ensure sufficient resources are available to serve load while satisfying transmission network and contingency constraints at least cost. In many restructured markets commitments are determined day-ahead (i.e. the day before the commitments are to take place) with a planning horizon of 24 single-hour periods. Some markets have different planning horizons or period lengths in their commitment process. The proposed market redesign for the California ISO will consist of a day-ahead unit commitment and a separate one-day commitment for the second day out. The purpose of this second unit commitment is to give advanced warning to units with extremely long start times that they will be needed to be online. The Electricity Pool in the original British market had a single-day planning horizon, but the commitment model consisted of 48 half-hour-long periods. The simulations and discussion in this section will assume the more standard planning horizon of 24 single-hour periods.

### 3.1. Centrally Committed Model

In centrally committed markets, the SO typically forecasts transmission network constraints for the following day, solicits demand bids from consumers, self-schedules and virtual bids, and generation offers consisting of costs and operating constraints.  

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<tr>
<th>Market</th>
<th>Commitment</th>
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<tr>
<td>Australia</td>
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<tr>
<td>Brazil</td>
<td>Self</td>
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<td>British</td>
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<td>NordPool</td>
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<td>PJM</td>
<td>Central</td>
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<td>Current design</td>
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<td>Texas Nodal redesign</td>
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Table 1: Examples of centralized versus decentralized unit commitment in selected electricity markets

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20 In some markets, the SO uses its own load forecasts instead of soliciting bids from consumers in order to ensure a sufficient set of units is committed. Many markets which do commit units on the basis of submitted demand bids have a separate reliability unit commitment process into which the SO substitutes its own load forecast and solves to find if additional units (beyond those selected in the initial unit commitment) must be started to ensure its load forecast can be satisfied.

21 Some centrally committed SOs give generators the option to self-commit their units and submit self-schedules and one-part energy offers as opposed to having the SO make commitment decisions for them. Most generators in such markets have tended to use the centralized commitment with multipart offers, instead. One possible reason for this behavior is that a generator which self-schedules a unit must recover all of its variable generating and fixed startup and no load costs through energy payments, and will have to roll these fixed costs into its one-part energy offers. Units which are centrally committed by the SO, by contrast, are given supplemental ‘make-whole’ payments if energy payments do not cover all of their costs. This different treatment of self-scheduled and centrally committed units can serve to make a self-committed unit less attractive in the SO’s dispatch, since the energy cost of the unit will seem higher. Make whole payments are described further later in this section.

22 SOs which include virtual bids in computing the unit commitment will often remove these bids in computing the reliability unit commitment, since virtual bids do not generally require actual physical delivery of energy. This is to ensure that there are sufficient units available when the virtual bids are netted out in real-time.
For ease of problem formulation, an SO which centrally commits units accepts costs and operating constraints as a set of standardized parameters. Most SOs accept three-part costs, consisting of startup, no-load, and marginal generating costs. Startup costs can usually be declared as being time-dependent with, for instance, a different startup cost when a unit is in a hot, intermediate, or cold state, with the state being dependent on how long a unit has been offline. Marginal generating costs are normally submitted as non-decreasing step or piecewise-linear functions. Operating constraints are similarly given as a set of parameters. For example, a ramp rate indicating the maximum allowable hourly change in a unit’s output, minimum and maximum output when online, minimum up and down times when a unit is started or stopped, etc. Although most unit costs and constraints can be parameterized in this manner, some SO commitment models are too rigid to fully capture unit characteristics. Combined-cycle gas turbines, for instance, have ‘sawtooth’-shaped cost functions because they can switch between different operating modes as their output changes. Cascaded hydroelectric systems have constraints linking the reservoirs within the watershed, which some SOs cannot fully capture in their standardized constraint parameters.

Given these inputs, the SO then determines a least-cost commitment and dispatch of the units to serve load while satisfying all operating, security, and ancillary service constraints. The SO will typically settle the market based on a uniform marginal energy price for each hour in the planning horizon at each location in the transmission network. These hourly locational marginal prices (LMPs) are paid to each unit that is dispatched to generate in that period. Because most units incur fixed startup and no-load costs, these linear energy-only payments can be confiscatory—meaning that inframarginal energy rents may not ensure that a generator recovers its full costs. Since such confiscation could incent generators to withhold themselves from the commitment process or to misstate costs or operating constraint parameters in their offers, potentially affecting the efficiency of the resulting commitment, centrally committed market typically include a supplemental ‘make-whole’ provision.

This make-whole provision pays any revenue shortfall over the course of the planning horizon to ensure committed units recover their stated costs. Units which receive energy payments in excess of their fixed and generating costs do not receive any supplemental make-whole payments. Although the make-whole provision ensures units recover their stated costs over the course of the day, a unit may still be run at a loss in a single hour. This could occur, for example, if a unit is kept online and generating at its minimum load (for instance because of a minimum up-time constraint), in which case it would not set the LMP and could be running at a marginal loss.

For ease of analysis and discussion the unit commitment formulation used in the simulations is a simplification of a commercial SO model. It includes stepped marginal costs and fixed (not time-dependent) startup and no-load costs for each unit. Demand is price-inelastic, there are no network flow constraints, virtual bids, self-schedules, or ancillary service requirements. The technical appendix discusses in further detail the exact formulation studied. The computations assume the centrally committed market settles with a uniform hourly energy price—specifically the dual variable associated with the hourly load balance constraints. As noted before, because linear energy-only payments can be confiscatory, the analysis further assumes that the SO includes a make-whole provision, which pays each unit the difference between its total costs incurred

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23 Streiffert et al. (2005) further note that because the SO’s commitment problem does not decouple when the load balance and ancillary service constraints are relaxed, unit commitment problems solved by an LR algorithm cannot easily model combined-cycle gas turbines or cascaded hydroelectric systems. Instead, they must often rely on other approximation algorithms.

24 A unit can also run at a marginal loss if it has any binding intertemporal (e.g. ramping) constraints and is the highest marginal cost unit, in which case it may not set the LMP. However, the LMPs used by the SO are typically computed from a separate optimal power flow (OPF) problem, which determines the optimal dispatch of the units given the fixed commitments from the unit commitment. Because these OPFs are solved separately for each hour, they do not include intertemporal constraints, thus a unit will only run at a marginal loss if it is held at minimum load.

25 As noted before, some SOs use LMPs based on the load balance constraint dual variables from the OPF, which can be different from the prices from the unit commitment load balance constraint dual variables. When there are no network flow constraints, the OPF LMPs will simply be set by the highest marginal cost unit, which is not running at minimum load. The unit commitment prices, on the other hand, will ‘smooth-out’ prices when high-cost units have binding intertemporal ramping constraints, and can in general be different. Because the two sets of prices are nearly identical in these simulations, the analysis uses only the unit commitment prices.
in the commitment and dispatch (calculated on the basis of costs stated in its offer) and the total energy payments received over the course of the 24 hours, if that difference is positive. These payments ensure that total net profits (on the basis of stated costs) are always non-negative.

3.2. Self-Committed Model

In a self-committed market, generators are left to make commitment decisions individually as opposed to the SO making binding commitment decisions for them. The exact format and means of energy trade in self-committed markets varies, however. The current Texas market, for instance, is based almost exclusively on bilateral transactions between generators and consumers. The original California market operated as a day-ahead energy-only market into which generators would offer their generation and are given energy payments only for their actual generation. The California market made dispatch decisions solely on the basis of the generators’ energy bids by aggregating them into a supply curve and intersecting them with demand, without any regard for operating constraints or startup or no-load costs. These constraints and fixed costs were meant to be internalized by generators, which would be compensated for energy generated only.\(^26\)

The model analyzed assumes that energy is traded through an energy-only market, such as the original California market design. To compute a competitive benchmark, the market is modeled as a competitive auction in which the auctioneer\(^27\) announces a set of hourly energy prices and price-taking generators individually determine their hourly commitments and output level to maximize profits and submit offers to the auctioneer indicating how many MWh they are willing to supply in each hour. The auctioneer then iteratively adjusts the hourly energy prices until it finds a set of prices which incent sufficient generation to serve the load. This iterative price-updating process is meant to mimic the proposal in Wilson (1997) for a self-committed market with two important differences. One is that loads are fixed in each hour as opposed to being price-elastic. Thus the market is assumed not to accept demand bids but rather solicit sufficient generation at any price to serve a fixed hourly load. The other is that under Wilson’s proposal, generators are assumed to submit offers consisting of quantity/price pairs. Because the model analyzed in this section assumes generators to behave competitively, generators are modeled as price-takers, which take the auction prices as fixed and decide their commitments and generation offers to maximize profits individually, as opposed to strategically adjusting their energy offers to raise energy prices.

Although the model assumes that energy is traded through a centralized energy market, it can be thought of as solving for a competitive equilibrium of direct bilateral trade between generators and consumers a la a Walrasian auction model. The model further assumes that the auctioneer starts with a set of prices which incent sufficient generation to serve the load, and iteratively adjust prices until finding a set of supporting minimal prices—which is a set of prices such that generators offer sufficient energy to serve the load, but would no longer do so if any of the energy prices were reduced. As the energy prices are dropped, higher-cost units will no longer find it profitable to commit themselves and the total quantity offered for generation will be driven towards the system load.

One difficulty with finding a set of supporting minimal prices is that the binary nature of the generators’ commitment decisions means that a set of supporting minimal prices will generally not be market-clearing, meaning generators will offer more total generation than there is load to serve, yet reducing any of the energy prices will cause a unit to decommit itself leaving insufficient energy to serve the load.\(^28\) One solution is to assume that the auction uses some type of rationing rule to determine how the load is divided amongst generators willing to commit themselves. The model assumes, instead, that because higher-cost generators drop out of the commitment as prices are iteratively reduced, if there is excess generation offered and multiple units are competing for the same load, the one with the lowest average cost over the course of the day will prevail.

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\(^26\)Generators could also offer capacity into separate ancillary service markets in which they could be paid for committed reserve capacity.

\(^27\)The auction can be thought of as being operated by the SO, or it can be a separate outside market.

\(^28\)This stems from the fact that if the SO unit commitment problem is solved using an LR algorithm, the dual solution will have a non-zero duality gap.
In modeling generators’ profit-maximizing behavior, they are assumed to perfectly rationally expect the behavior of other generators and take into account the ‘winner determination assumption’ in making their own commitment decisions. This is to preclude the possibility that a unit may commit itself in expectation of being dispatched but finds that it doesn’t, resulting in a net profit loss. This assumption is enforced algorithmically, and described along with the formulation and solution algorithm in further detail in the technical appendix. Finally, the model assumes that each generator acts independently in making its commitment decisions, as opposed to making commitment decisions for portfolios of generators being owned by generating firms. This assumption is made because the dataset used does not have unit ownership information, although the technique and results would translate to a setting with generator asset portfolios.

3.3. Market Simulations

The simulations are based on actual market data from an ISO New England (ISONE) unit commitment problem in February of 2005, consisting of 276 dispatchable units. This dataset is used because of availability of the data, and is meant to be an illustrative example of the relative efficiency losses and settlement costs between the two market designs. Figure 1 shows the ISONE system map and the 9 major pricing zones. The ISONE system covers approximately 6.5 million retail customers, includes more than 350 generators with 31,000 MW of installed capacity, an all-time peak load of 28,127MWh, and $11 billion of annual energy trade.

The operating costs and constraint parameters used in the market simulations are those which were submitted by generators to ISONE. The competitive benchmark assumption takes these generator-offered parameters as reflecting actual costs and unit operating constraints—thereby assuming away any incentive compatibility issues. The computation of the central commitment assumes generators will offer these actual cost and constraint parameters to the SO for use in its commitment problem, as opposed to strategically misstating them to increase profits. The computation of the self-commitment assumes that generators behave as price-takers and maximize profits with the same cost and constraint parameters.

Table 2 compares the total settlements paid to generators, commitment costs, and profits of the generators in the simulations of the two market designs. Although the centrally committed market is assumed to include a make-whole provision, the dataset is such that each generator receives sufficient inframarginal rents to recover all its costs and no supplemental payments are required. Nonetheless, Figure 2 shows that the set of supporting minimal prices found in the self-committed market far exceed the energy prices paid in the central unit commitment.

Indeed, a critical assumption underlying a centrally committed market is that the SO can force cross subsidies of “losing” hours by profits from other hours and has means of preventing generators from making adjustments to their assigned schedules. Figure 3 shows the resulting load imbalances which would occur if generators could individually adjust their outputs to maximize profits against the hourly energy prices—known as uninstructed deviations. Due to the potential for such deviations, SOs penalize such deviations in generation by requiring generators to buy or sell back their insufficient or excess generation at the locational marginal price (LMP), thereby removing any incentive for such deviations.

This enforcement mechanism can be problematic, however, in multiple-settlement systems in which the SO computes different sets of LMPs at different intervals in real-time. Because there can be differences between the prices at which an uninstructed deviation is paid and penalized, a generator may be inclined to change its output if these price differences are predictable. Alternatively, O’Neill et al. (2005) propose a two-part nonlinear tariff, which prevents such deviations. Their proposed scheme pays the same uniform LMPs29 and a set of discriminatory payments on each of the commitment variables.30 O’Neill et al demonstrate that such a pricing scheme will ensure that profit-maximizing generators will follow the same centrally-determined

29Importantly, these LMPs must be the prices from the unit commitment problem, not from the associated OPFs. Moreover, the pricing scheme enforces the unit commitment dispatch, not that from the OPF. If the SO dispatches according to the OPF, then those OPF constraints must be incorporated into the unit commitment formulation.
30These commitment variable payments are found by solving the unit commitment problem to optimality, adding constraints to fix the values of the commitment variables, solving the linear programming relaxation of the unit commitment problem, and using the dual variables on the constraints fixing the commitment variables.
commitment and schedule if they could adjust their outputs, without the need for any additional penalty or enforcement mechanism.\footnote{Their results apply much more broadly to any market with integer variables or other nonconvexities.} Inclusion of such a provision in our simulation would increase total settlement costs of the centrally committed market by approximately $219,017, which is a relatively small sum compared to the total simulated settlement costs of the market of approximately $16 million.

<table>
<thead>
<tr>
<th>Market Design</th>
<th>Energy Payments</th>
<th>Make-Whole Payments</th>
<th>Total Settlements</th>
<th>Commitment Costs</th>
<th>Total Unit Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central</td>
<td>$16,075,120.54</td>
<td>$0.00</td>
<td>$16,075,120.54</td>
<td>$5,758,200.97</td>
<td>$10,316,919.57</td>
</tr>
<tr>
<td>Self</td>
<td>$25,060,666.38</td>
<td>$25,060,666.38</td>
<td>$6,003,274.22</td>
<td>$19,057,392.17</td>
<td></td>
</tr>
<tr>
<td>%-difference</td>
<td>55.89722%</td>
<td>4.25607%</td>
<td></td>
<td>84.71979%</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Cost and profit comparison of centrally and self-committed market designs

More importantly, the simulation demonstrates that a self-committed market requires higher energy prices than a centrally committed one. Because the model assumes that demand is fixed and inelastic, these higher prices are simply a wealth transfer from consumers to generators, without any efficiency losses. In a more realistic setting with demand response,\footnote{See Zarnikau (2008) for a discussion of demand response.} the higher prices could result in allocative distortions.

Table 2 also shows that a self-committed market will generally suffer productive efficiency losses, as demonstrated by the more than 4% increase in total commitment and dispatch costs. These efficiency losses are not the result of units committing themselves under a self-committed market when they would not be committed under the central commitment. Rather these losses stem from the fact that a central commitment gives the most efficient coordination of generator dispatches, which are lost when generators dispatch themselves independently. Of the 276 units, 108 are committed in at least one hour under the central unit commitment solution. Of these 108, 73 follow the same commitment and dispatch schedule under the self-committed market as under the central unit commitment, with some shuffling of generation amongst the remaining 35.
4. Conclusions

This paper has revisited one of the key issues surrounding the proper role of the SO in the design of competitive electricity markets. On one hand an SO with broad economic authority can, in theory, determine the most efficient commitment to meet forecasted demand. However, centrally committed markets, are not strategy-proof and are prone to incentive compatibility issues, meaning that generators can profitably manipulate their offers to increase profits. This has both been shown through simple examples and was one criticism of the original Electricity Pool in the British market. Proponents claim that a decentralized energy-only market in which generators individually determine their commitments can reduce the incentive issues of a central unit commitment while minimizing efficiency losses. The simulation of a competitive benchmark conducted in this paper provides a bound on the productive efficiency losses from a self- as opposed to centrally committed market design.

While these losses were relatively small, around 4.25% in the case examined here, this would nonetheless represent a significant welfare loss in absolute terms considering that ISO markets typically trade energy worth billions of dollars on an annual basis. The efficiency loss in the market simulations would amount to an annual loss of nearly $90 million for the ISO system, if the results are typical of most days.\footnote{This may likely be a lower-bound on the annual losses, since more energy would presumably be traded during summer peak periods.} Moreover, the total efficiency losses would be significantly higher in the presence of demand response since the energy prices under self-commitment will generally be higher than under central commitment. These higher prices could cause self-committed markets to be more prone to allocative efficiency losses in the presence of demand response, which may be an important consideration as these programs are slowly becoming more prevalent.
Although these simulations are based on a dataset from a single SO for a specific period of time, the results are nonetheless instructive for estimating the relative size of the efficiency losses between the two market designs. The issue of unit commitment is an important and often overlooked one. Many authors have been quick to advocate one market design or another, all while suppressing these important technical realities of power systems. The discussion and analysis of this paper and the issues raised would be relevant to any market that is evaluating the options of centralized versus decentralized design of day-ahead markets. In some sense, the options available to policy makers and market engineers is between two imperfect systems, since centralized markets will be fraught with incentive problems and decentralized markets with coordination losses.

A. Technical Appendix

This section describes the specific formulations and algorithms used in modeling of the centrally and self-committed market.

A.1. Central Commitment Model

To describe the formulation, the following notation is used:

Problem Parameters

- \( I \): generator index set,
- \( T \): number of planning periods,
- \( B \): number of steps in generators’ marginal cost
- \( SU_i \): startup cost of unit \( i \),
- \( N_i \): no-load cost of unit \( i \),
- \( MC^b_i \): marginal generating cost of step \( b \) of unit \( i \)’s marginal cost curve,
- \( \kappa_b^i \): maximum generating capacity of step \( b \) of unit \( i \)’s marginal cost curve,
- \( K^-_{i,t} \): minimum generating capacity of unit \( i \) in period \( t \),
- \( K^+_{i,t} \): maximum generating capacity of unit \( i \) in period \( t \),
- \( R_i \): maximum ramp rate of unit \( i \),
- \( n_i \): minimum up time of unit \( i \),
- \( f_i \): minimum down time of unit \( i \),
- \( D_t \): load forecast in period \( t \).

Decision Variables

- \( q^b_{i,t} \): generation provided from step \( b \) of unit \( i \) in period \( t \),
- \( u_{i,t} \): binary variable indicating if unit \( i \) is up in period \( t \),
- \( s_{i,t} \): binary variable indicating if unit \( i \) is started in period \( t \), and
- \( h_{i,t} \): binary variable indicating if unit \( i \) is stopped in period \( t \).
The problem is then formulated as minimizing total commitment costs:

$$\min_{q,u,s,h} \sum_{i,t} \left( \sum_{b} MC_b^b q_{i,t}^b + N_i u_{i,t} + SU_i s_{i,t} \right);$$

subject to load-balance:

$$\sum_{i,b} q_{i,t}^b = l_t, \forall t;$$

unit generating capacity:

$$K^-_{i,t} u_{i,t} \leq \sum_{b} q_{i,t}^b \leq K^+_{i,t} u_{i,t}, \forall i, t;$$

unit segment capacity:

$$0 \leq q_{i,t}^b \leq \kappa^b_{i}, \forall i, t;$$

unit ramping limit:

$$-R_i \leq \sum_{b} (q_{i,t}^b - q_{i,t-1}^b) \leq R_i, \forall i, t;$$

unit minimum up time:

$$\sum_{\tau=t-n_i+1}^{t} s_{i,\tau} \leq u_{i,t}, \forall i, t, b;$$

unit minimum down time:

$$\sum_{\tau=t-f_i+1}^{t} h_{i,\tau} \leq 1 - u_{i,t}, \forall i, t;$$

startup definition:

$$s_{i,t} \geq u_{i,t} - u_{i,t-1}, \forall i, t;$$

unit shutdown definition:

$$h_{i,t} \geq u_{i,t-1} - u_{i,t}, \forall i, t;$$

and integrality:

$$u_{i,t}, s_{i,t}, h_{i,t} \in \{0, 1\}, \forall i, t.$$

### A.2. Self-commitment Model

The self-commitment model assumes that the market operates as an iterative energy-only auction. The auctioneer announces a set of hourly energy prices, given by $p_t$. Generators then individually determine their profit-maximizing commitments and dispatches and submit to the auctioneer a set of offers indicating how many MWh of energy they are willing to generate in each of the hours. The auctioneer then iteratively adjusts the prices until reaching a set of supporting minimal prices.

The ‘winner determination rule,’ which states that if two or more generators are contending for the same dispatch the one with the lowest average cost will prevail, is enforced algorithmically. The model assumes that generators, in making their commitment and dispatch decision, will be perfectly rational in predicting other generators’ behavior and the winner determination rule outcome. This is to preclude the possibility that a generator commits itself thinking that it will be dispatched, but finding that it is not because of the commitment decision of another unit with a lower average cost. This is enforced in each generator’s profit-maximization problem by restricting each generator to produce no more than the available load in each hour, where the available load in each hour is the total load in that hour less the dispatch of lower average cost units. As units are accepted for dispatch in order of average cost, the available load is updated, and the profit-maximization problem of the remaining generators are iteratively resolved.
Using the same notation as from section A.1, define \( p_t \) to be the hourly energy prices announced by the auctioneer and \( a_t \) to be the available load in each hour. Generator \( i \)'s profit-maximization problem is then formulated as maximizing profits:

\[
\mathcal{P}_i : \max_{q,u,s,h} \sum_t \left[ \sum_b (p_t - MC_{ib}) q_{ib,t} - N_i u_{i,t} - SU_i s_{i,t} \right];
\]

subject to load availability:

\[
\sum_b q_{ib,t} \leq a_t, \quad \forall \, t;
\]

and the remaining unit operating constraints:

\[
K_{i,t} u_{i,t} \leq \sum_b q_{ib,t} \leq K_{i,t}^+ u_{i,t}, \quad \forall \, t; \\
0 \leq q_{ib,t} \leq q_{ib}^\text{p}, \quad \forall \, t,b; \\
-R_i \leq \sum_b (q_{ib,t} - q_{ib,t-1}) \leq R_i, \quad \forall \, t; \\
\sum_{\tau=t-n_i+1}^t s_{i,\tau} \leq u_{i,t}, \quad \forall \, t; \\
\sum_{\tau=t-f_i+1}^t h_{i,\tau} \geq 1 - u_{i,t}, \quad \forall \, t; \\
s_{i,t} \geq u_{i,t} - u_{i,t-1}, \quad \forall \, t; \\
h_{i,t} \geq u_{i,t-1} - u_{i,t}, \quad \forall \, t; \\
u_{i,t}, s_{i,t}, h_{i,t} \in \{0, 1\}, \quad \forall \, t, b.
\]

For ease of notation, we define \( \sigma_i = (q_i, u_i, s_i, h_i) \) to be generator \( i \)'s schedule. We then define \( C_i(\sigma_i) = \sum_t \left( \sum_b MC_{ib} q_{ib,t} + N_i u_{i,t} + SU_i s_{i,t} \right) \) to be generator \( i \)'s total cost and \( Q_i(\sigma_i) = \sum_t q_{ib,t} \) to be generator \( i \)'s total generation when following the schedule given by \( \sigma_i \). Given a set of energy prices, \( p_t \), we then find the resulting dispatch using the following algorithm:

\[
\text{initialize available load } \quad a_t \leftarrow D_t \quad \forall \, t \\
\text{set each generator's status to uncommitted } \quad \text{dispatch}_i \leftarrow 0 \quad \forall \, i \\
\text{repeat} \\
\hspace{1em} \forall \, i \text{ s.t. } \text{dispatch}_i = 0 \text{ solve } \mathcal{P}_i \\
\hspace{1em} \{ \text{solve each uncommitted generator's profit-maximization problem} \} \\
\hspace{1em} \forall \, i \text{ s.t. } \text{dispatch}_i = 0 \text{ let } \text{avgcost}_i \leftarrow \frac{C_i(\sigma_i)}{Q_i(\sigma_i)} \\
\hspace{1em} \{ \text{compute each uncommitted generator’s average cost} \} \\
\hspace{1em} j \leftarrow \arg \min_i \{ \text{avgcost}_i : \text{dispatch}_i = 0, Q_i(\sigma_i) > 0 \} \\
\hspace{1em} \{ \text{find lowest average cost uncommitted unit, denote as } j \} \\
\hspace{1em} \text{fix commitment and dispatch of unit } j \\
\hspace{1em} \text{dispatch}_j \leftarrow 1 \\
\hspace{1em} a_t \leftarrow a_t - \sum_b q_{jb,t} \quad \forall \, t = 1, \ldots, T \\
\hspace{1em} \{ \text{update available load based on dispatch of unit } j \} \\
\text{until } a_t = 0 \quad \forall \, t = 1, \ldots, T \text{ or } \text{dispatch}_i = 1 \quad \forall \, i \in I \\
\{ \text{repeat until all load served or all units dispatched} \}
\]

The algorithm is designed to find a set of profit-maximizing commitments and dispatches, while also ensuring that generators properly take into account how much load will be available for them to supply in each hour given the behavior of other generators and the winner determination assumption. It does so by iteratively accepting generators one at a time in order of their average costs, updating the load available in each hour based on the dispatch of the accepted units, and resolving the profit-maximization problems of
the uncommitted units. The process continues until all the load is served, in which case the energy prices are called a feasible set of supporting prices, or until no other generators wish to commit themselves, in which case the energy prices do not incent sufficient commitment to serve the load.

To find a set of supporting minimal prices, the auction begins with a set of supporting prices that will induce oversupply. The prices are then iteratively updated to be $p_t \leftarrow \alpha p_t + (1 - \alpha) \eta_t$, where $\eta_t$ is the highest average cost of the units dispatched to generate in period $t$ and $\alpha \in (0, 1)$ is a step-size parameter. In doing so, the auction aims to drive the energy price in each hour towards the marginal average cost. Once the prices cannot be feasibly updated in this manner, the auction decreases the energy price in each period individually so as to minimize the total settlement costs paid by the SO.

References


